Questions

1. Measurement, 2UG 2011 HSC 4 MC

The angle of depression from a kookaburra’s feet to a worm on the ground is 40°. The worm is 15 metres from a point on the ground directly below the kookaburra’s feet.

How high above the ground are the kookaburra’s feet, correct to the nearest metre?

(A) 10 m
(B) 11 m
(C) 13 m
(D) 18 m
Danni is flying a kite that is attached to a string of length 80 metres. The string makes an angle of 55° with the horizontal.

How high, to the nearest metre, is the kite above Danni’s hand?

(A) 46 m
(B) 66 m
(C) 98 m
(D) 114 m
3. Measurement, 2UG 2010 HSC 10 MC

A plane flies on a bearing of $150^\circ$ from $A$ to $B$.

What is the bearing of $A$ from $B$?

(A) $30^\circ$

(B) $150^\circ$

(C) $210^\circ$

(D) $330^\circ$
Two trees on level ground, 12 metres apart, are joined by a cable. It is attached 2 metres above the ground to one tree and 11 metres above the ground to the other.

What is the length of the cable between the two trees, correct to the nearest metre?

(A) 9 m  
(B) 12 m  
(C) 15 m  
(D) 16 m
5. Measurement, 2UG 2012 HSC 10 MC

What is the area of this triangle, to the nearest square metre?

(A) 33 m²
(B) 37 m²
(C) 42 m²
(D) 44 m²

6. Measurement, 2UG 2010 HSC 9 MC

Three towns \( P \), \( Q \) and \( R \) are marked on the diagram.

The distance from \( R \) to \( P \) is 76 km. \( \angle RQP = 26^\circ \) and \( \angle RPQ = 46^\circ \)

What is the distance from \( P \) to \( Q \) to the nearest kilometre?

(A) 100 km
(B) 125 km
(C) 165 km
(D) 182 km
Town $B$ is $80$ km due north of Town $A$ and $59$ km from Town $C$. Town $A$ is $31$ km from Town $C$.

What is the bearing of Town $C$ from Town $B$?

(A) $019^\circ$

(B) $122^\circ$

(C) $161^\circ$

(D) $341^\circ$
8. Measurement, 2UG 2008 HSC 5 MC

What is the size of the smallest angle in this triangle?

(A) 29°
(B) 47°
(C) 58°
(D) 76°


In the diagram, $AD$ and $DC$ are equal to 30 cm.

What is the length of $AB$ to the nearest centimetre?

(A) 28 cm
(B) 31 cm
(C) 34 cm
(D) 39 cm
What is the value of $\theta$, to the nearest degree?

(A) 21°
(B) 32°
(C) 43°
(D) 55°
The diagram shows the position of $Q$, $R$ and $T$ relative to $P$.

In the diagram,

- $Q$ is SW of $P$
- $R$ is NW of $P$
- $\angle QPT$ is $165^\circ$

What is the bearing of $T$ from $P$?

(A) $060^\circ$
(B) $075^\circ$
(C) $105^\circ$
(D) $120^\circ$

A point $P$ lies between a tree, 2 metres high, and a tower, 8 metres high. $P$ is 3 metres away from the base of the tree.

From $P$, the angles of elevation to the top of the tree and to the top of the tower are equal.

What is the distance, $x$, from $P$ to the top of the tower?

(A) 9 m  
(B) 9.61 m  
(C) 12.04 m  
(D) 14.42 m
The following information is given about the locations of three towns $X$, $Y$ and $Z$:

- $X$ is due east of $Z$
- $X$ is on a bearing of $145^\circ$ from $Y$
- $Y$ is on a bearing of $060^\circ$ from $Z$.

Which diagram best represents this information?
14. Measurement, 2UG 2013 HSC 26a

Triangle $PQR$ is shown.

![Diagram of triangle PQR with sides 53 m, 98 m, and 66 m](image)

Find the size of angle $Q$, to the nearest degree. \((2 \text{ marks})\)

15. Measurement, 2UG 2009 HSC 23a

The point $A$ is 25 m from the base of a building. The angle of elevation from $A$ to the top of the building is $38^\circ$.

![Diagram of building with angle 38° and distances](image)

(i) Show that the height of the building is approximately 19.5 m. \((1 \text{ mark})\)

(ii) A car is parked 62 m from the base of the building.

What is the angle of depression from the top of the building to the car? Give your answer to the nearest degree. \((2 \text{ marks})\)
16. Measurement, 2UG 2010 HSC 24d

The base of a lighthouse, $D$, is at the top of a cliff 168 metres above sea level. The angle of depression from $D$ to a boat at $C$ is $28^\circ$. The boat heads towards the base of the cliff, $A$, and stops at $B$. The distance $AB$ is 126 metres.

(i) What is the angle of depression from $D$ to $B$, correct to the nearest degree? (3 marks)

(ii) How far did the boat travel from $C$ to $B$, correct to the nearest metre? (2 marks)

17. Measurement, 2UG 2010 HSC 26d

Find the area of triangle $ABC$, correct to the nearest square metre. (3 marks)
A compass radial survey of the field $ABCD$ has been conducted from $O$.

Find the area of the section $ABO$, to the nearest square metre. (2 marks)
A yacht race follows the triangular course shown in the diagram. The course from $P$ to $Q$ is 1.8 km on a true bearing of 058°.

At $Q$ the course changes direction. The course from $Q$ to $R$ is 2.7 km and $\angle PQR = 74°$.

(i) What is the bearing of $R$ from $Q$? (1 mark)

(ii) What is the distance from $R$ to $P$? (2 marks)

(iii) The area inside this triangular course is set as a 'no-go' zone for other boats while the race is on.

What is the area of this 'no-go' zone? (1 mark)
20. Measurement, 2UG 2011 HSC 24c

A ship sails 6 km from A to B on a bearing of $121^\circ$. It then sails 9 km to C. The size of angle $ABC$ is $114^\circ$.

Copy the diagram into your writing booklet and show all the information on it.

(i) What is the bearing of C from B? (1 mark)

(ii) Find the distance AC. Give your answer correct to the nearest kilometre. (2 marks)

(iii) What is the bearing of A from C? Give your answer correct to the nearest degree. (3 marks)


A disability ramp is to be constructed to replace steps, as shown in the diagram. The angle of inclination for the ramp is to be $5^\circ$.

Calculate the extra distance, $d$, that the ramp will extend beyond the bottom step. Give your answer to the nearest centimetre. (3 marks)
22. Measurement, 2UG 2008 HSC 25c

Pieces of cheese are cut from cylindrical blocks with dimensions as shown.

Twelve pieces are packed in a rectangular box. There are three rows with four pieces of cheese in each row. The curved surface is face down with the pieces touching as shown.

(i) What are the dimensions of the rectangular box? \( (4 \text{ marks}) \)

To save packing space, the curved section is removed.

(ii) What is the volume of the remaining triangular prism of cheese? Answer to the nearest cubic centimetre. \( (2 \text{ marks}) \)
23. Measurement, 2UG 2014 HSC 28b

A radial compass survey of a sports centre is shown in the diagram.

(i) Show that the size of angle $AOB$ is $114^\circ$. (1 mark)

(ii) Calculate the length of the boundary $AB$, to the nearest metre. (2 marks)

(iii) Find the area of triangle $AOB$ in hectares, correct to two significant figures. (3 marks)
Raj cycles around a course. The course starts at $E$, passes through $F$, $G$ and $H$ and finishes at $E$. The distances $EH$ and $GH$ are equal.

(i) What is the length of $EF$, to the nearest kilometre?  \(2\) marks

(ii) What is the total distance that Raj cycles, to the nearest kilometre?  \(3\) marks
1. Measurement, 2UG 2011 HSC 4 MC

∠ Elevation (worm) = 40° (alternate angles)

\[
\tan 40° = \frac{h}{15}
\]

\[ h = 15 \times \tan 40° \]
\[ = 12.58... \text{ m} \]

⇒ C


\[
\sin 55° = \frac{h}{80}
\]

\[ h = 80 \times \sin 55° \]
\[ = 65.532... \text{ m} \]

⇒ B
3. Measurement, 2UG 2010 HSC 10 MC

\[ \angle TBA = 30^\circ \quad (180^\circ \text{ in } \Delta) \]

\[ \Rightarrow \text{Bearing of } A \text{ from } B \]
\[ = 360 - 30 \]
\[ = 330^\circ \]
\[ \Rightarrow D \]


Using Pythagoras

\[ c^2 = 12^2 + 9^2 \]
\[ = 144 + 81 \]
\[ = 225 \]
\[ \Rightarrow c = 15, \ c > 0 \]
\[ \Rightarrow C \]
5. Measurement, 2UG 2012 HSC 10 MC

Let unknown angle = $\angle C$

$\angle C = 180 - (50 + 57) \ (180^{\circ} \text{ in } \Delta)$

$= 73^{\circ}$

Using $A = \frac{1}{2}ab \sin C$

$\therefore A = \frac{1}{2} \times 9.9 \times 8.8 \times \sin 73^{\circ}$

$= 41.656 \ \text{m}^2$

$\Rightarrow C$

6. Measurement, 2UG 2010 HSC 9 MC

$\angle QRP = 180 - (26 + 46) \ (180^{\circ} \text{ in } \Delta)$

$= 108^{\circ}$

Using sine rule

$\frac{PQ}{\sin 108^{\circ}} = \frac{76}{\sin 26^{\circ}}$

$PQ = \frac{76 \times \sin 108^{\circ}}{\sin 26^{\circ}}$

$= 164.88 \ \text{km}$

$\Rightarrow C$

7. Measurement, 2UG 2012 HSC 20 MC

Using the cosine rule

$\cos \angle B = \frac{a^2 + c^2 - b^2}{2ac}$

$= \frac{59^2 + 80^2 - 31^2}{2 \times 59 \times 80}$

$= 0.9449...$

$\angle B = 19^{\circ} \ (\text{nearest degree})$

$\therefore \ \text{Bearing of Town C from } B = 180 - 19 = 161^{\circ}$

$\Rightarrow C$
8. Measurement, 2UG 2008 HSC 5 MC

Smallest angle is opposite smallest side.

Using \( \cos A = \frac{b^2 + c^2 - a^2}{2bc} \)

\[
\cos A = \frac{7^2 + 8^2 - 6^2}{2 \times 7 \times 8} = 0.6875
\]

\( \therefore A = 46.567...^\circ \)

\( \Rightarrow B \)


\( \triangle ADC \) is isosceles

\( \angle DAB = \angle DCA = x^\circ \)

\( \Rightarrow 2x + 80^\circ = 180^\circ \) (180° in \( \triangle ADC \))

\( 2x = 100^\circ \)

\( x = 50^\circ \)

\( \angle DBA = 180^\circ - (50 + 60) \) (180° in \( \triangle ADB \))

\( = 70^\circ \)

Using sine rule

\[
\frac{AB}{\sin 60^\circ} = \frac{30}{\sin 70^\circ}
\]

\( AB = \frac{30 \times \sin 60^\circ}{\sin 70^\circ} = 27.648... \text{ cm} \)

\( \Rightarrow A \)
10. Measurement, 2UG 2013 HSC 24 MC

Using \( \frac{a}{\sin A} = \frac{b}{\sin B} \)

\[
\frac{82}{\sin A} = \frac{100}{\sin 26}
\]

\[\sin A = \frac{82 \times \sin 26}{100} = 0.35946... \]

\[\angle A = 21^\circ \text{ (nearest degree)} \]

Since \(180^\circ\) in \(\Delta\)

\[90 + 26 + (\theta + 21) = 180\]

\[\theta = 43^\circ \]

\[\Rightarrow C\]

11. Measurement, 2UG 2008 HSC 17 MC

\[\angle QPS = 45^\circ \text{ (Q is south west of } P)\]

\[\angle TPS = 165 - 45 = 120^\circ \]

\[\therefore \angle NPT = 60^\circ\text{ (180° in straight line)}\]

\[\Rightarrow A\]

Triangles are similar (2 $\angle$s same)
In smaller triangle

\[ h^2 = 2^2 + 3^2 \]
\[ h^2 = 4 + 9 \]
\[ h = \sqrt{13} \]

\[ \frac{x}{8} = \frac{\sqrt{13}}{2} \] (sides of similar $\triangle$s same ratio)

\[ x = \frac{8\sqrt{13}}{2} \]
\[ x = 14.422... \]

$\Rightarrow D$


Since $X$ is due east of $Z$

$\Rightarrow$ Cannot be $B$ or $D$

The diagram shows we can find

$\angle ZYX = 60 + 35^\circ = 95^\circ$

Using alternate angles ($60^\circ$) and

the $145^\circ$ bearing of $X$ from $Y$

$\Rightarrow C$
Using cosine rule

\[ \cos \angle Q = \frac{a^2 + b^2 - c^2}{2ab} \]
\[ = \frac{53^2 + 66^2 - 98^2}{2 \times 53 \times 66} \]
\[ = -0.3486... \]

\[ \therefore \angle Q = 110.4034... \]
\[ = 110^\circ \text{ (nearest degree)} \]
15. Measurement, 2UG 2009 HSC 23a

(i) Need to prove height \( h \approx 19.5 \) m

\[
\tan 38^\circ = \frac{h}{25}
\]

\[
h = 25 \times \tan 38^\circ
\]

\[
= 19.5321...
\]

\[
\approx 19.5 \text{ m } \text{... as required.}
\]

(ii)

![Diagram of elevation and depression angles](image)

Let \( \angle \text{ Elevation (from car)} = \theta \)

\[
\tan \theta = \frac{h}{62}
\]

\[
= \frac{19.5}{62}
\]

\[
= 0.3145...
\]

\[
\theta = 17.459...^\circ
\]

\[
= 17^\circ \text{ (nearest degree)}
\]

\[
\therefore \angle \text{ Depression to car} = 17^\circ \text{ (alternate to } \theta)\]

♦♦ Mean mark 33%

MARKER’S COMMENT:
Students who didn’t round to the nearest degree lost a mark here.
16. Measurement, 2UG 2010 HSC 24d

(i) \( \tan \angle ADB = \frac{126}{168} \)
\( \angle ADB = 36.8698... \)
\( = 36.9^\circ \) (to 1 d.p)
\( \angle \text{Depression } D \text{ to } B = 90 - 36.9 \)
\( = 53.1 \)
\( = 53^\circ \) (nearest degree)

(ii) Need to find \( CB \)
\( \angle ADC + 28 = 90 \)
\( \angle ADC = 62^\circ \)
\( \tan 62^\circ = \frac{AC}{168} \)
\( AC = 168 \times \tan 62^\circ \)
\( = 315.962... \)
\( CB = AC - AB \)
\( = 315.962... - 126 \)
\( = 189.962... \)
\( = 190 \text{ m (nearest m)} \)
17. Measurement, 2UG 2010 HSC 26d

\[
\cos \angle C = \frac{AC^2 + CB^2 - AB^2}{2 \times AC \times CB}
\]
\[
= \frac{50^2 + 40^2 - 83^2}{2 \times 50 \times 40}
\]
\[
= -0.69725...
\]

\[\angle C = 134.2067\ldots^\circ\]

Using Area = \(\frac{1}{2}ab \sin C\)

Area \(\Delta ABC\) = \(\frac{1}{2} \times 50 \times 40 \times \sin 134.2067\ldots^\circ\)

= 716.828...

= 717 m² (nearest m²)

18. Measurement, 2UG 2013 HSC 28a

\[\angle AOB = 21^\circ + (360 - 310)\]

= 71°

Using Area = \(\frac{1}{2}ab \sin C\)

Area = \(\frac{1}{2} \times 60 \times 75 \times \sin 71^\circ\)

= 2127.4167...

= 2127 m² (nearest m²)
19. Measurement, 2UG 2009 HSC 27b

(i)

\[ \angle PQS = 58^\circ \text{ (alternate to } \angle TPQ) \]

Bearing of \( R \) from \( Q \)

\[ = 180^\circ + 58^\circ + 74^\circ \]

\[ = 312^\circ \]

(ii) Using cosine rule

\[ RP^2 = RQ^2 + PQ^2 - 2 \times RQ \times PQ \times \cos \angle RQP \]

\[ = 2.7^2 + 1.8^2 - 2 \times 2.7 \times 1.8 \times \cos 74^\circ \]

\[ = 7.29 + 3.24 - 2.679... \]

\[ = 7.851... \]

\[ \therefore RP = \sqrt{7.851...} \]

\[ = 2.8019... \]

\[ \approx 2.8 \text{ km (1 d.p.)} \]

(iii) Using \( A = \frac{1}{2}ab \sin C \)

\[ A = \frac{1}{2} \times 2.7 \times 1.8 \times \sin 74^\circ \]

\[ = 2.3358... \]

\[ = 2.3 \text{ km}^2 \]

\[ \therefore \text{ No-go zone is 2.3 km}^2 \]
20. Measurement, 2UG 2011 HSC 24c

(i) Let point $D$ be due North of point $B$

\[ \angle ABD = 180 - 121 \text{ (cointerior with } \angle A) \]
\[ = 59^\circ \]
\[ \angle DBC = 114 - 59 \]
\[ = 55^\circ \]
\[ \therefore \text{ Bearing of } C \text{ from } B \text{ is } 055^\circ \]

(ii) Using cosine rule

\[ AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \times \cos\angle ABC \]
\[ = 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos 114^\circ \]
\[ = 160.9275... \]
\[ \therefore AC = 12.685... \text{ (Noting } AC > 0) \]
\[ = 13 \text{ km (nearest km)} \]

(iii) Need to find $\angle ACB$ (see diagram)

\[ \cos\angle ACB = \frac{AC^2 + BC^2 - AB^2}{2 \times AC \times BC} \]
\[ = \frac{(12.685...)^2 + 9^2 - 6^2}{2 \times (12.685...) \times 9} \]
\[ = 0.9018... \]
\[ \angle ACB = 25.6^\circ \text{ (to 1 d.p.)} \]
From diagram
\[ \angle BCE = 55^\circ \text{ (alternate to } \angle DBC) \]
\[ \therefore \text{ Bearing of } A \text{ from } C \]
\[ = 180 + 55 + 25.6 \]
\[ = 260.6 \]
\[ = 261^\circ \text{ (nearest degree)} \]


Let the horizontal part of the ramp = \( x \) cm

\[ \tan 5^\circ = \frac{39}{x} \]
\[ x = \frac{39}{\tan 5^\circ} \]
\[ = 445.772\ldots \]

Since \( x = 60 + d \)
\[ d = 445.772 - 60 \]
\[ = 385.772 \text{ cm} \]
\[ = 386 \text{ cm (nearest cm)} \]

22. Measurement, 2UG 2008 HSC 25c

(i) Box height = 15 cm
(radius of the arc)

Box width = \( 3 \times 7 \)
\[ = 21 \text{ cm} \]

Box length = \( 4x \)
Using cosine rule
\[ c^2 = a^2 + b^2 - 2ab \cos C \]
\[ x^2 = 15^2 + 15^2 - 2 \times 15 \times 15 \times \cos 40^\circ \]
\[ = 450 - 344.7199... \]
\[ = 105.2800... \]
\[ x = 10.2606... \]
\[ \Rightarrow \text{Box length} = 4 \times 10.2606... \]
\[ = 41.04... \]
\[ \therefore \text{Dimensions are } 41 \text{ cm} \times 21 \text{ cm} \times 15 \text{ cm} \]

(ii) Volume = \( Ah \)
\[ h = 7 \text{ cm} \]
Need to find \( A \)

Using \( A = \frac{1}{2} ab \sin C \)
\[ \Rightarrow A = \frac{1}{2} \times 15 \times 15 \times \sin 40^\circ \]
\[ = 72.3136... \]
\[ \therefore V = 72.3136... \times 7 \]
\[ = 506.195... \]
\[ = 506 \text{ cm}^3 \text{ (nearest whole)} \]
Let $D$ be directly north of $O$

$\angle AOD = 360 - 320 = 40^\circ$

$\therefore \angle AOB = 40 + 74 = 114^\circ \ldots$ as required

(ii) Using cosine rule

$AB^2 = AO^2 + BO^2 - 2 \times AO \times BO \times \cos \angle AOB$

$= 287^2 + 211^2 - 2 \times 287 \times 211 \times \cos 114^\circ$

$= 126890 - 121114(-0.4067\ldots)$

$= 176151.50\ldots$

$AB = 419.704\ldots$

$= 420\text{ m (nearest m)}$

(iii) Using $A = \frac{1}{2}ab \sin C$

Area $\triangle AOB = \frac{1}{2} \times 287 \times 211 \times \sin 114^\circ$

$= 27660.786\ldots \text{ m}^2$
Mean mark 48%.

Mean mark 37%.

MARKER’S COMMENT: Students could also have used Pythagoras or the Sine rule to calculate GH.

24. Measurement, 2UG 2012 HSC 29c

(i) Need to find $EF$

\[ \angle FGE = 180 - (139 + 31) \quad (180^\circ \text{ in } \Delta) \]

\[ = 180 - 170 \]

\[ = 10^\circ \]

Using sine rule

\[ \frac{EF}{\sin 10^\circ} = \frac{82}{\sin 139^\circ} \]

\[ EF = \frac{82 \times \sin 10^\circ}{\sin 139^\circ} \]

\[ = 21.70406 \ldots \]

\[ = 22 \text{ km (nearest km)} \]

(ii) Let $d =$ total distance cycled

Need to find $EH$

Since $\triangle EGH$ is isosceles, and $\angle EHG = 90^\circ$

\[ \angle GEH = \angle HGE = 45^\circ \quad (\text{equal angles opp. equal sides}) \]

\[ \Rightarrow \sin 45^\circ = \frac{GH}{82} \]

\[ GH = 82 \times \sin 45^\circ \]

\[ = 57.983 \ldots \]

\[ d = EF + FG + GH + EH \]

\[ = 21.704 \ldots + 64 + 57.983 + 57.983 \]

\[ = 201.66 \ldots \]

\[ = 202 \text{ km (nearest km)} \]

\[ = 27.660 \ldots \text{ ha} \quad (1 \text{ ha} = 10,000 \text{ m}^2) \]

\[ = 28 \text{ ha (2 sig figures)} \]